# ELEN E3106/4106 Lecture 6

**Diffusion of Carriers**

Outline

- Drift current (loose ends)
- Diffusion processes
- Diffusion & drift
- Built-in fields
- Diffusion with recombination
- Continuity equation & diffusion length

**Assignments:**

Reading: Streetman and Banerjee §4.4.1-4.4.4

Homework 2 due Friday Sept 19th by 5pm

# Back to Basics: Relationship between Drift Current and Resistance

- Recall from Lecture ~~4~~, 5,

Resistance

$$\boxed{R = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma}}$$

conductivity

$$\boxed{\sigma = qn\mu_n + qp\mu_p}$$

drift velocity

$$\boxed{v_n = -\mu_n E}$$

- Where usually only __majority__ carrier component dominates conductivity

- How can we back out current? Let's imagine we have a n-type material:

Ohm's law: __V = IR__ $\rightarrow I = \frac{V}{R} = \frac{V(wt\sigma)}{L} = \frac{VA(qn\mu_n)}{L} = qAn(\mu_n E) = qAnv_n$


A = wt

# Recap of carrier motion/currents (so far!)

- Recall from Lecture 4, we learned that there is random motion (Brownian) of $e^-$ and $h^+$ in the absence of E-field
  - Net motion of carriers = 0
  - E-field = 0, but T > 0, carriers move with thermal velocity,

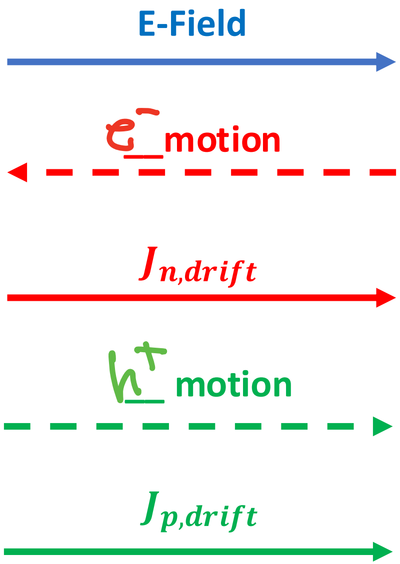
$$v_{th} = \sqrt{\frac{3kT}{m^*}}$$

- In the presence of an E-field, carriers will have **drift** velocity, $v_d = \pm \mu E$
- Where the carrier mobility (ease with which carriers move in semi) is,

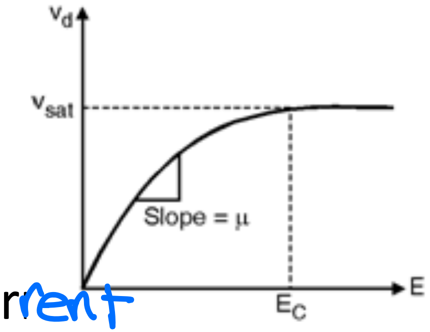
$$\mu = -\frac{q\tau_C}{m^*} \quad \frac{cm^2}{V\cdot s}$$

- So the net current in the presence of an E-field is,

$$J_{drift} = J_{n,drift} + J_{p,drift}$$



# High Field Effects on Drift Velocity




**Velocity saturation**

- Recall from last lecture,

$$\boxed{J_n^{drift} = -qnv_{dn} = qn\mu_n E} \quad \boxed{J_p^{drift} = +qpv_{dp} = qp\mu_p E}$$

- Which '=' sign do we want?
  - The first '=' sign always applies, and we can find the drift current
    - $J_{n,drift} = -qnv_{dn}$
    - $J_{p,drift} = qpv_{dp}$
- At low E-fields, Ohm's law is valid (current density is directly proportional to electric field), and the second '=' sign applies:
  - We call this the ohmic regime,
    - $J_{n,drift} = qn\mu_n E$
    - $J_{p,drift} = qp\mu_p E$
- At high E-fields, ($\geq E_c$, or $E_{crit}$ — critical E-field), electrons will reach **drift** saturation velocity and exhibit a sublinear dependence on the electric field (e.g. we use first '=' sign because $v_{d,sat} \neq \mu E$ )

Sources: Electronics-Tutorial.net

# Problem: Drift Current Calculations

n-type!

- A 2 cm long piece of Si with cross-sectional area of 0.1 cm$^2$ is doped with donors at $10^{15}$ cm$^{-3}$, and has a resistance of 90 Ω. The saturation velocity of electrons in Si is $10^7$ cm/s for fields above $10^5$ V/cm. Calculate the electron drift velocity, if we apply a voltage of 100 V across the piece. What is the current through the piece if we apply a voltage of $10^6$ V across it?

$\ll E_{crit} = 10^5$ V/cm

- First, we need to find the electric field,
- $E = \frac{\text{Voltage across semiconductor}}{\text{Length of semiconductor}} = \frac{100\ V}{2\ cm} = \underline{50}\ V/cm$ --> Which regime are we in? ohmic regime
- We can estimate the mobility from Figure 3-23 and solve,
- $v_d = \mu_n E = \left(\underline{1500}\ \frac{cm^2}{V \cdot s}\right)\left(50\ \frac{V}{cm}\right) = 75000\ cm/s$

1500 cm²/V-s

- Now for the larger electric field,
- $E = \frac{10^6\ V}{2\ cm} = 5 \times 10^5\ V/cm$ --> > $E_{crit}$ → saturation regime
- Using current equation from slide 2,
- $I = qAnv_n = (1.6 \times 10^{-19} C)(0.1\ cm^2)(10^{15}\ cm^{-3})\left(10^7\ \frac{cm}{s}\right) = 160\ A$

$= 160$ A

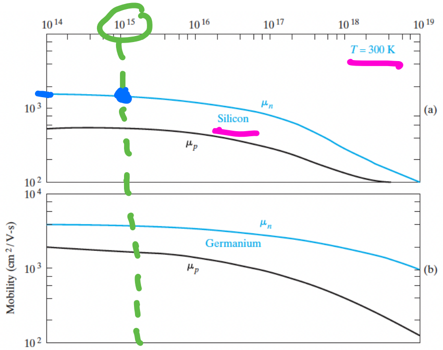



**Figure 3-23 in textbook. Use for extrinsic semiconductors. Note you should use the total # impurities (Na+Nd)**

Sources: Textbook

# Diffusion

- In addition to drift, there is a second component to current called diffusion

- Particles move from point of higher concentration to a point of lower concentration

- What are some other real world examples of diffusion processes?
  - Coffee aroma wafting through the air
  - Soy sauce in wonton soup

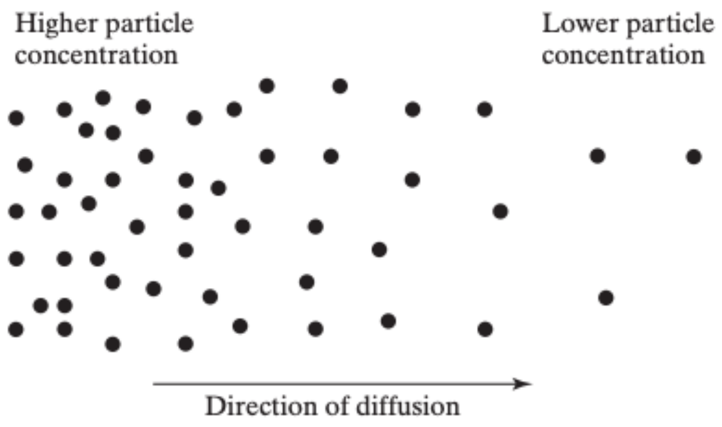


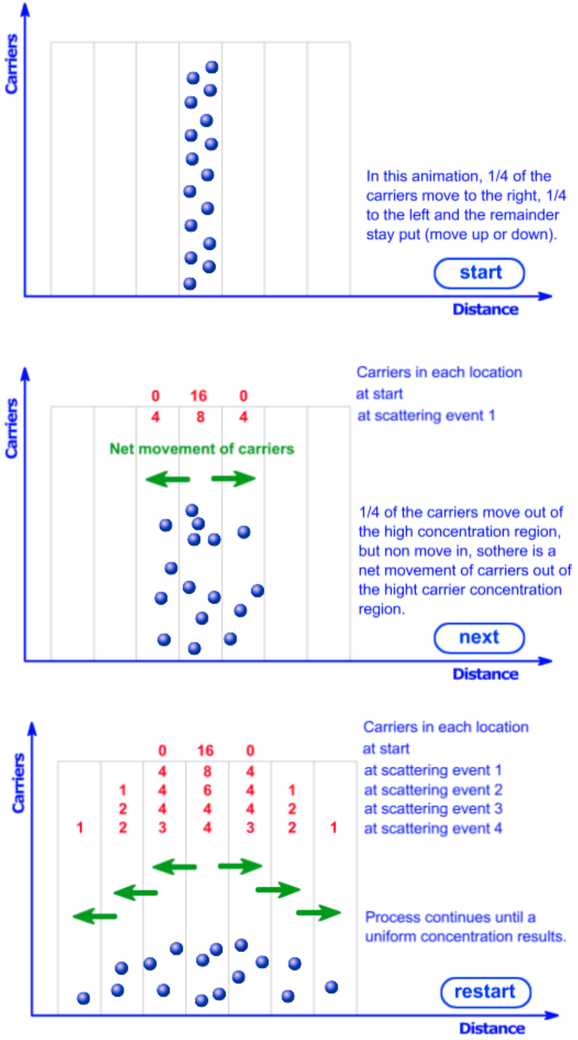

Figure 1

Diffusion

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits,* School Physics, Science Prof Online

# What drives diffusion processes?

- Carriers undergo random thermal motion and collide with each other, and scatter
- They are moving in arbitrary directions until they collide, and move in new directions
- Carriers will have net thermal motion from high to low concentration areas
- When does diffusion stop? Once there is no longer a concentration gradient (e.g. the concentration is uniform)




Sources: https://www.pveducation.org/pvcdrom/pn-junctions/diffusion#:~:text=Diffusion%20is%20the%20random%20scattering,in%20cm2s%2D1.

# Diffusion Current

- What drives the net diffusion current?

  the concentration gradient

- Will we have diffusion current in a uniform sample?

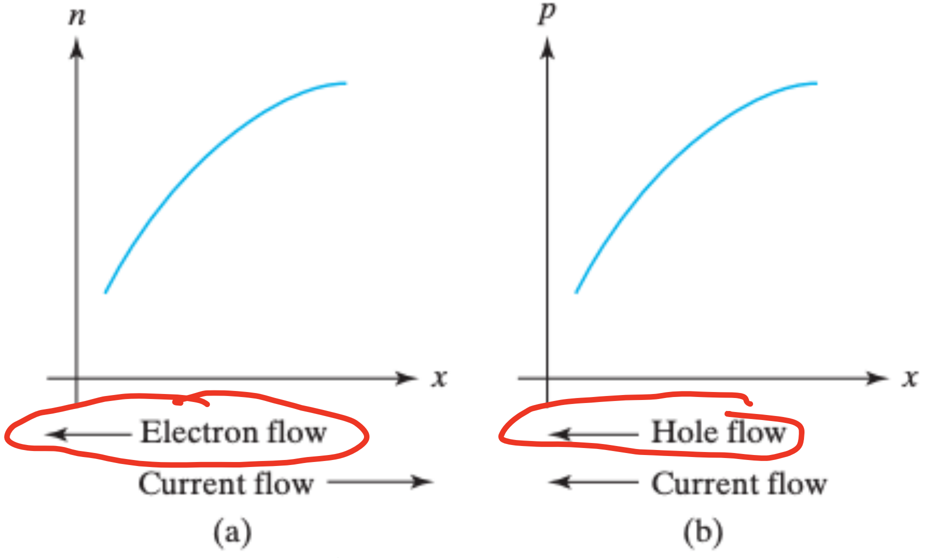
  No!

- The rate of diffusion is proportional to the concentration gradient,

$$J_{n,\,\text{diffusion}} \propto \frac{\mathrm{d}n}{\mathrm{d}x}$$

- And for holes?

$$|J_{p,\,\text{diffusion}}| \propto \left|\frac{dp}{dx}\right|$$




Currents in same direction!

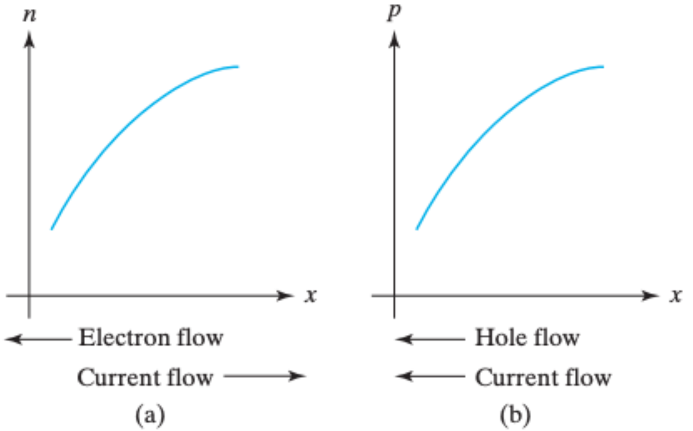
Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

# Diffusion Current Equations and Coefficients

- Mathematically, we can write the diffusion current densities,

(-)

$$\boxed{J_{n,\text{diffusion}} = qD_n\frac{dn}{dx}} \qquad \boxed{J_{p,\text{diffusion}} = -qD_p\frac{dp}{dx}}$$



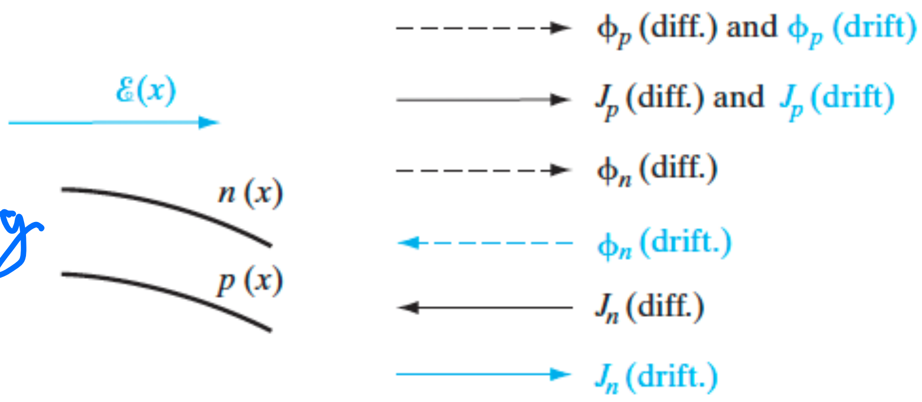

- $D_n$ and $D_p$ are the diffusion coefficients
  - Units? cm²/s
- Why the (-) sign? The net motion of e- due to diffusion is in the direction of decreasing e⁻ concentration, so derivative is (-)dn/dx. (-q)(-dn/dx) --> we get (+)

- Are the diffusion currents in the same direction? No! See plots above

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

# Total Currents and Visualizing Particle Motion

- In general in semiconductors, there can be 4 possible current sources:
    1. Electron drift
    2. Electron diffusion
    3. Hole drift
    4. Hole diffusion




- For electrons: sum current density

$$\boxed{J_n = J_{n,\text{drift}} + J_{n,\text{diffusion}} = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}}$$

- For holes:

$$\boxed{J_p = J_{p,\text{drift}} + J_{p,\text{diffusion}} = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx}}$$

**Dashed lines ($\phi$) denote direction of particle motion. Solid lines denote the resulting current direction**

- And we can write the total current as the sum,

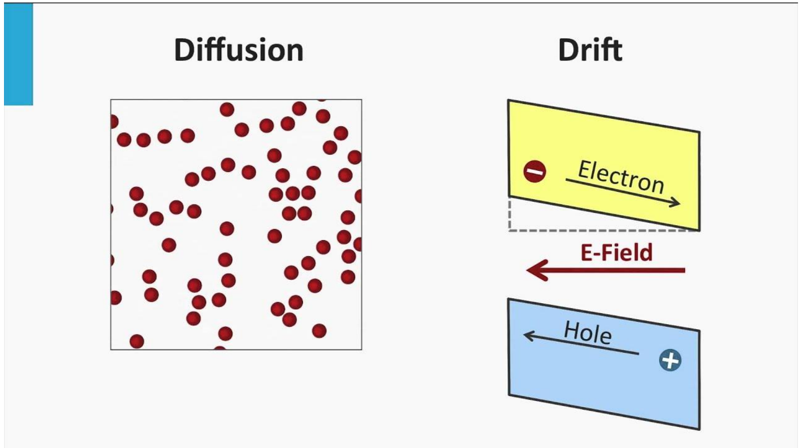
$$J = J_n + J_p$$

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

# The Influence of Majority Carriers on Diffusion

- Important: Minority carriers rarely contribute much to drift current (there are too few of them),

$$J_n = J_{n,\text{drift}} + J_{n,\text{diffusion}} = qn\mu_n\mathscr{E} + qD_n\frac{dn}{dx}$$




- BUT if their gradient is high enough….

  minority carrier currents (diffusion) can be high

- Result: minority carrier diffusion currents can sometimes be as large as majority carrier currents

Sources: E. Pop ECE 340 Slides, A. Smets TU Delft Lecture Week 2

# Built-In Fields

- Under equilibrium, open-circuit conditions, the total current must always be __0__
- No net current flows (I.e. $J_{drift} = - J_{diffusion}$)
- So any disturbance (e.g. light, doping gradient, thermal gradient) which may set up a carrier concentration gradient will also internally set up a built-in _E-field__
- What does this tell us? There must be some relationship between diffusion and drift. We can set the earlier total current equation equal to zero,

$$J_n = J_{n,\mathrm{drift}} + J_{n,\mathrm{diffusion}} = qn\mu_n\mathscr{E} + qD_n\frac{\mathrm{d}n}{\mathrm{d}x} = 0$$

Sources: E. Pop ECE 340 Slides, A. Smets TU Delft Lecture Week 2

# The Einstein Relation between $D$ and $\mu$

- Solving the equation (noting the equilibrium Fermi level does not vary with x, and the derivative of $E_i$ as given in the textbook Eq. 4-26), we get:

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$

$D_n, \mu_n$ $D_p, \mu_p$

- This is called the Einstein relationship and is valid for both $e^-$ and $h^+$
- What does this allow us to calculate?

Either $D$ or $\mu$ from measurement of the other

- This relation (almost) always holds true
- Physically, all scattering mechanisms (e.g. phonon/lattice and impurity scattering) that impede carrier drift also impede carrier diffusion.

Table 4–1 Diffusion coefficient and mobility of electrons and holes for intrinsic semiconductors at 300 K. *Note:* Use Fig. 3–23 for doped semiconductors.

| | $D_n$ (cm$^2$/s) | $D_p$ (cm$^2$/s) | $\mu_n$ (cm$^2$/V-s) | $\mu_p$ (cm$^2$/V-s) |
|---|---|---|---|---|
| Ge | 100 | 50 | 3900 | 1900 |
| Si | 35 | 12.5 | 1350 | 480 |
| GaAs | 220 | 10 | 8500 | 400 |

Sources: Textbook

# Relationship between Energy Diagrams, Voltage, and E-field

- When a voltage is applied across a piece of semiconductor, it alters the band diagrams

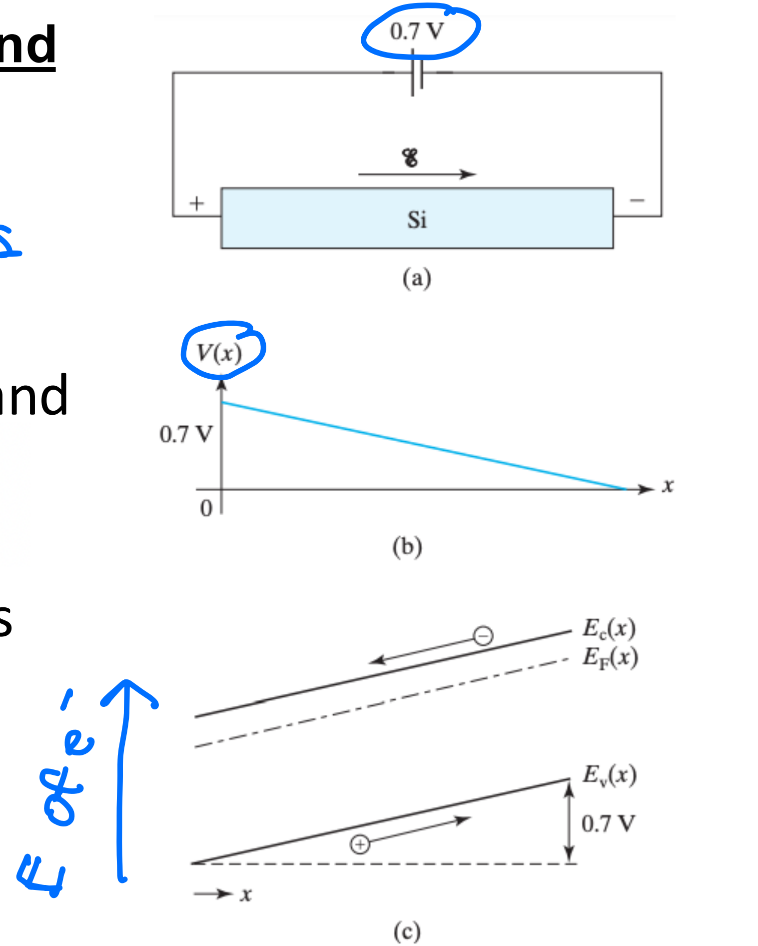
- (+) V raises the potential energy of a (+) charge and lowers the P.E. of a (-) charge

$$\mathcal{E}(x) = -\frac{dV(x)}{dx}$$

- Therefore, a (+) V lowers the energy diagrams since we are plotting the energy of an electrons (- charges)

- How do we convert from V to eV? Multiply by $q$

$$E_c(x) = \text{constant} - qV(x)$$




Energy band diagram of a semiconductor under applied voltage (+7V).

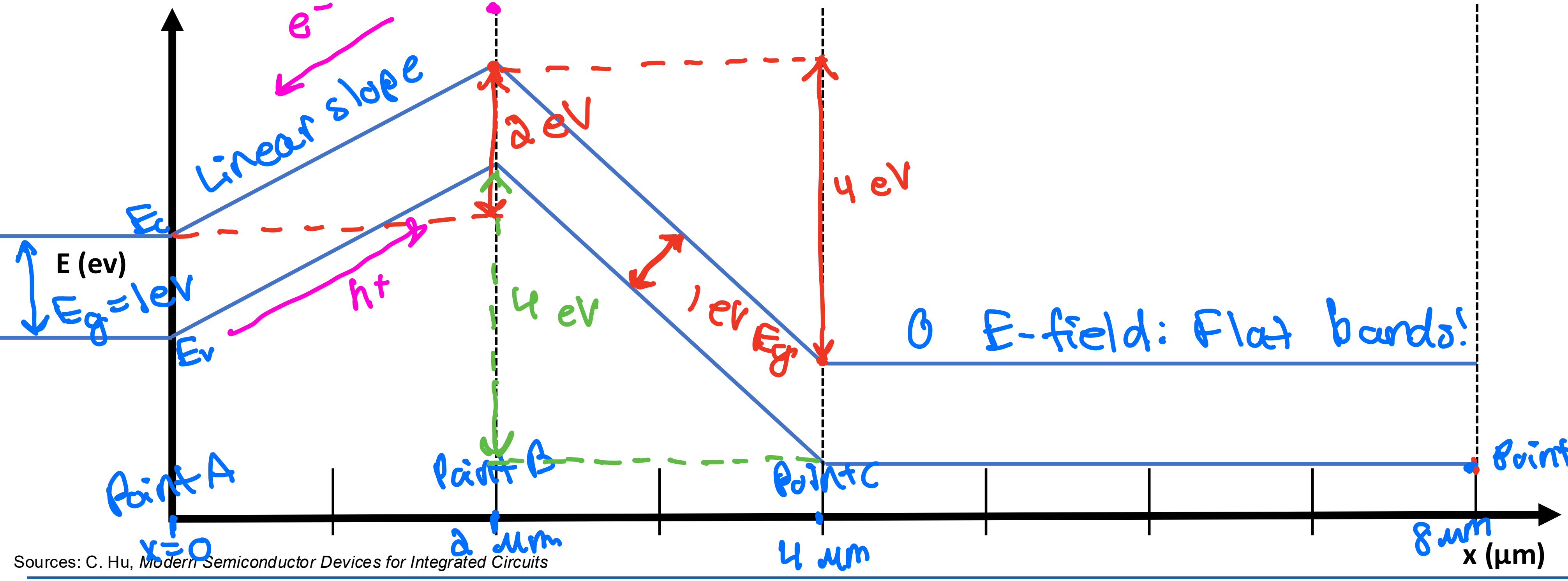
Sources: Textbook

# Problem: Sketching Energy Band Diagrams with Applied Voltage

A semiconductor has a bandgap of 1 eV. It is subjected to the following potentials at the various locations as follows (assume linear variation of potentials between locations):

- Point A at x = 0 µm, V = 0 V
- Point B at x = 2 µm, V = -2 V
- Point C at x = 4 µm, V = +4 V
- Point D at x = 8 µm, E-field is zero between C and D

*$E_F$ is not moving within $E_g$!



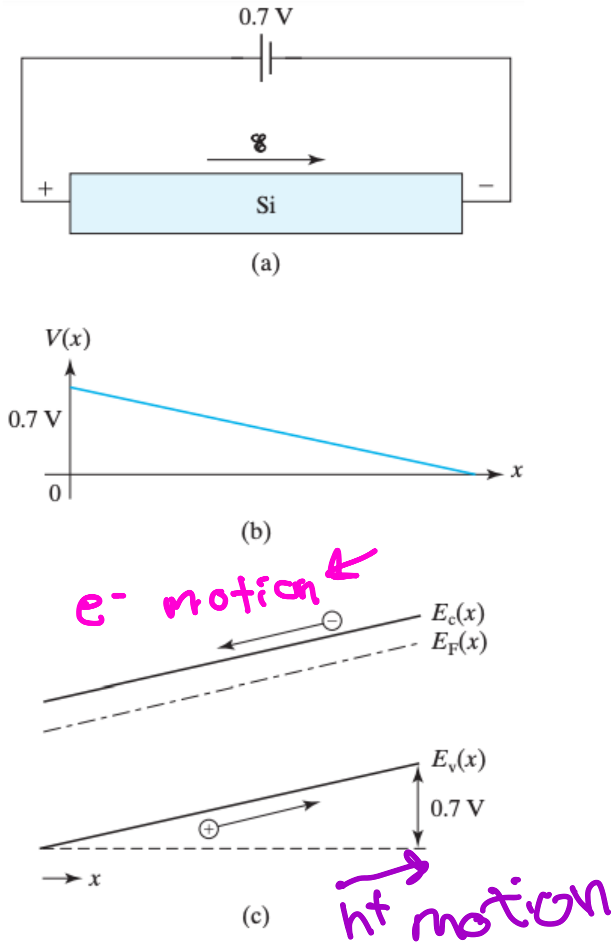

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

# Relationship between Energy Diagrams, Voltage, and E-field

- Practical points to note:
  - $E_c$ and $E_v$ are higher where the voltage is lower
  - The slope of $E_c$ and $E_v$ indicates the E-field

$$\mathscr{E}(x) \equiv -\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{1}{q}\frac{\mathrm{d}E_c}{\mathrm{d}x} = \frac{1}{q}\frac{\mathrm{d}E_v}{\mathrm{d}x}$$

- Useful analogies:
  - The e- will roll downhill in the energy band diagram
  - The h+ will float uphill
- Recall: both are seeking lowest energy state (edges of bands)



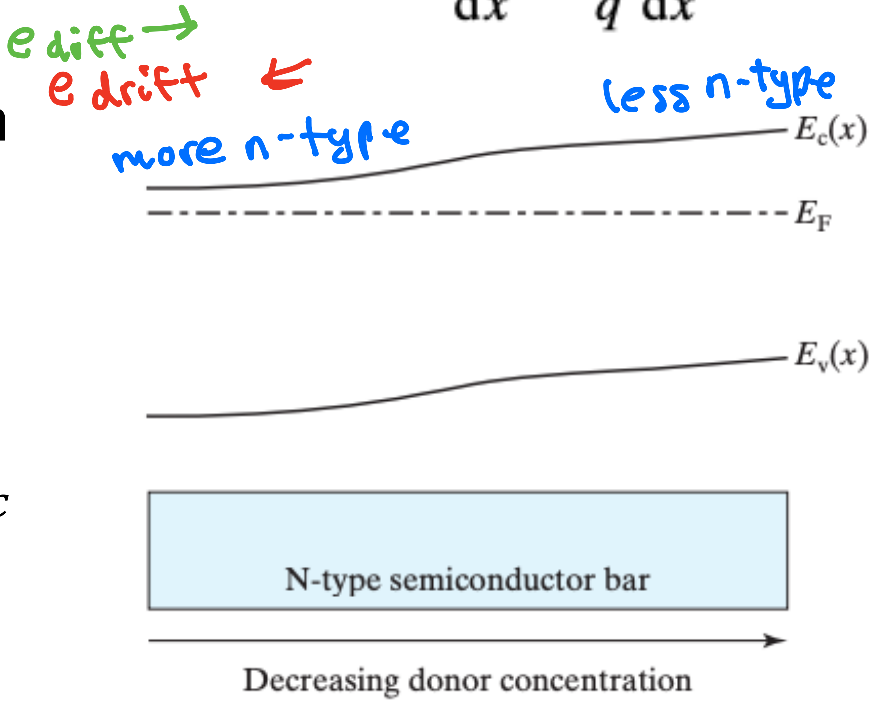

**Energy band diagram of a semiconductor under applied voltage (+7V).**

Sources: Textbook

# Looking at Built-In Fields Again

- Now that we understand the effects of voltage and fields on the band diagram, we can perhaps better understand the built-in fields that arise from concentration gradients in equilibrium

$$\mathscr{E}(x) \equiv -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx}$$




- In equilibrium: Fermi level is constant
- Left side more heavily doped than right: $E_c$ is closer to $E_F$
- Because $E_c$ is not constant, an E-field will be created as real as a field created by an external voltage

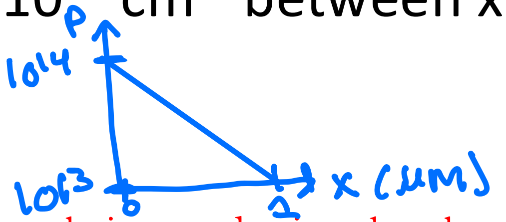
Sources: Textbook

# Problem: Calculating carrier diffusion

donors *Recall $n \approx N_D$, $p \approx N_A \rightarrow$ none!

The hole density in an n-type silicon wafer ($N_D = 10^{17}$ cm$^{-3}$) decreases linearly from $10^{14}$ cm$^{-3}$ to $10^{13}$ cm$^{-3}$ between x = 0 and x = 1 µm. Calculate the hole diffusion current.

density




- Rearranging the Einstein relation, and using the plot to estimate mobility,
  - $D_p = \frac{kT}{q}\mu_p = 0.026(317) = 8.2\,\frac{cm^2}{s}$
- What is dp/dx? Change in carrier concentration over change in distance:
  - $\frac{dp}{dx} = \frac{10^{14} - 10^{13}\ cm^{-3}}{10^{-4}\ cm}$
- Now we can use the diffusion current equation,
  - $J_{p,diffusion} = qD_p\frac{dp}{dx} = (1.62 \times 10^{-19}C)\left(8.2\,\frac{cm^2}{s}\right)\left(\frac{9\times10^{13}cm^{-3}}{10^{-4}cm}\right) =$ $1.18\ A/cm^2$

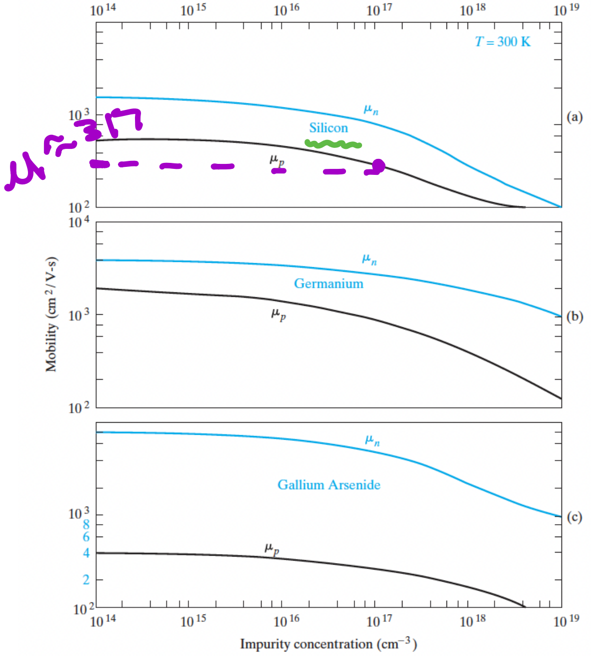



**Figure 3-23 in textbook. Use for extrinsic semiconductors. Note you should use the total # impurities (Na+Nd)**

Sources: Textbook

# Recap: Diffusion (without recombination)

- Diffusion without recombination is driven by the carrier concentration gradient
- We have the Einstein relation diff. → $\frac{D}{\mu} = \frac{kT}{q}$ ← drift
- We can look up the mobility from plots, but make sure you use the total impurity concentration ($N_A + N_D$)
- $kT/q$ at room temperature is ~0.026 V (you can memorize this, but be careful at temperatures different from 300K)

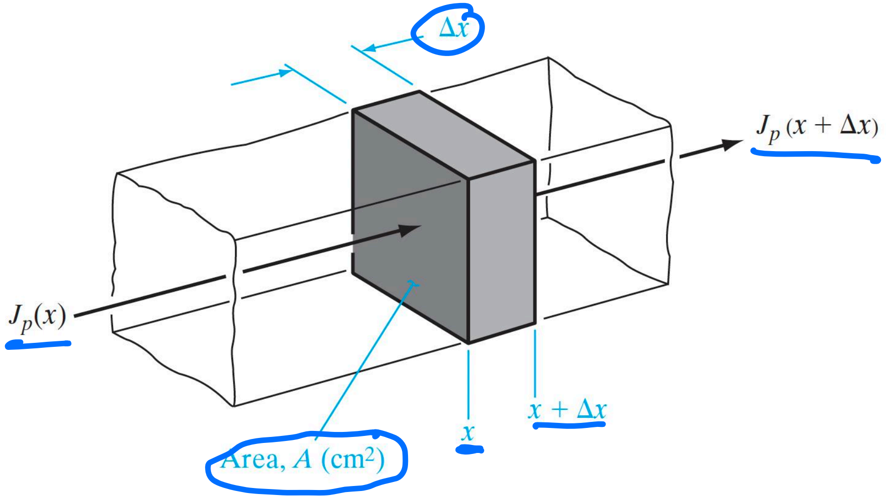
- But what about in the case where we have recombination effects?
  - Recall: excess EHPs can recombine

Sources: Textbook

# Diffusion with Recombination

- Recombination can change carrier concentrations, so we must consider the effects on diffusion
- Consider an n-type semi sample with area $A$ and a 'slice' of length $\Delta x$
- Minority current density entering the area is: $J_p(x)$
- Minority current density leaving the area is: $J_p(x+\Delta x)$
- Simple counting
  - Rate of hole population increase = (current IN – current OUT) – hole recombination

n-type




Sources: Textbook

# Diffusion with Recombination

1 C/s = 1 A

- So let's count holes("bubbles"):
  - Recombination rate = # excess bubbles (δp) / recombination time (τ)

    excess hole concentration (non-equilibrium)
  - Current (#bubbles) IN – Current (#bubbles) OUT = $J_{IN} - J_{OUT}$ / dx

$p = p_0 + \delta p$

equilibrium

- What are the units?
  - Recombination rate: holes/cm³/s
  - Current Density: holes/cm²/s
- Therefore, we must account for width dx (cm) of volume slice

$$\left.\frac{\partial p}{\partial t}\right|_{x \to x+\Delta x} = \frac{1}{q}\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

in time

Rate of hole buildup = increase of hole concentration in $\delta x A$ per unit time − recombination rate

Sources: Textbook

# Diffusion Length

- What does this mean in *steady-state*?
  - Distribution of excess carriers is maintained

Time derivatives go to zero

- The diffusion equation in steady-state:

$$\boxed{\begin{aligned} \frac{d^2\delta n}{dx^2} &= \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2} \\ \frac{d^2\delta p}{dx^2} &= \frac{\delta p}{D_p\tau_p} \equiv \frac{\delta p}{L_p^2} \end{aligned}}$$

← no time derivates, function of distance

- The diffusion length $L_p$ = $\sqrt{D_p \tau_p}$ is a figure of merit.
- $L_{p:}$ average length a carrier moves between generation and recombination

Sources: Textbook